

Mass spectra and decay properties of D Meson in a relativistic Dirac formalism

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ABSTRACT: The mass spectra of D meson states are calculated in the framework of a relativistic independent quark model. For the present study, we have used the martin like potential for the quark confinement. Our predicted states in S-wave, 2^3S_1 (2605.86 MeV) and 2^1S_0 (2521.72 MeV) are in very good agreement with experimental result of $2608 \pm 2.4 \pm 2.5$ MeV and $2539.4 \pm 4.5 \pm 6.8$ MeV respectively reported by BABAR Collaboration. The calculated P-wave D meson states, 1^3P_2 (2468.22 MeV), 1^3P_1 (2404.94 MeV), 1^3P_0 (2315.24 MeV) and 1^1P_1 (2367.94 MeV) are in close agreement with experimental average (Particle Data Group) values of 2462.6 ± 0.7 MeV, $2427 \pm 26 \pm 25$ MeV, 2318 ± 29 MeV and 2421.3 ± 0.6 MeV respectively. The pseudoscalar decay constant ($f_P = 202.57$ MeV) of D meson obtained using this relativistic formalism is in very good agreement with the experiment as well as with the lattice and other available theoretical predictions. The Cabibbo favoured hadronic decay branching ratios, $\text{BR}(D^0 \rightarrow K^- \pi^+)$ as 3.835% and $\text{BR}(D^0 \rightarrow K^+ \pi^-)$ as 1.069×10^{-4} are also in very good agreement with the respective experimental values of $3.91 \pm 0.08\%$ and $(1.48 \pm 0.07) \times 10^{-4}$ reported by CLEO Collaboration. Our predicted results in leptonic decay widths of D meson are also in better accord with experiment as well as other theoretical results. The mixing parameters of $D^0 - \bar{D}^0$ oscillation, x_q (5.14×10^{-3}), y_q (6.02×10^{-3}) and R_M (3.13×10^{-5}) are in very good agreement with BaBar and Belle Collaboration results.

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1 Introduction

Very recently, experiments at LHCb [1] have reported large number of D_J resonances in the mass range of $2.0\text{GeV}/c^2$ to $4.0\text{GeV}/c^2$ of which many of them belong to natural excited states of D meson while quite a number of them belong to unnatural states [1]. It is important and necessary to exhaust the possible conventional description of $q\bar{Q}$ excitations [2] before resorting to more exotic interpretations [3, 4]. Further theoretical efforts are still required in order to explain satisfactorily the recent experimental data concerning these open-charm states.

Apart from the challenges posed by the exotics, there are also many states which are admixtures of their nearby natural states. For example, the discoveries of new resonances of D states such as $D(2550)$ [6], $D(2610)$ [6], $D(2640)$ [7], $D(2760)$ [6] etc., have further generated considerable interest towards the spectroscopy of this open charm mesons. Study of D meson carry special interest as it is a hadron with two open flavours (c, \bar{u} or \bar{d}) which restricts its decay via strong interactions. These resonance states thus provide us a clean laboratory to study electromagnetic and weak interactions. The masses of low-lying $1S$ and $1P_J$ states of D mesons are recorded both experimentally [2] and theoretically [8–13]. Though lattice QCD and QCD sum rule are quite successful, but their predictions for the excited states of the open flavor mesons in the heavy sector are very few. However recent experimental data on excited $D-$ states are partially inconclusive and require more detailed analysis involving their decay properties. The understanding of the weak transition form

factors of heavy mesons is important for a proper extraction of the quark mixing parameters, for the analysis of non-leptonic decays and CP violating effects. QCD sum rule (QSR) [14–18] is non-perturbative approach to evaluate hadron properties by using the correlator of the quark currents over the physical vacuum and it is implemented with the operator product expansion (OPE). Lattice QCD (LQCD) [19–21] is also non-perturbative approach to use a discrete set of spacetime points (lattice) to reduce the analytically intractable path integrals of the continuum theory to a very difficult numerical computation. QCD sum rules are suitable for describing the low q^2 region of the form factors; lattice QCD gives good predictions for high q^2 . As a result these methods do not provide for a full picture of the form factors and more significant, for the relations between the various decay channels. Potential models provide such relations and give the form factors in the full q^2 -range.

Thus any attempts towards the understanding of these newly observed states become very important for our understanding of the light quark/antiquark dynamics within $q\bar{q}/Q\bar{Q}$ bound states. So, a successful theoretical model aims to provide important information about the quark-antiquark interactions and the behavior of QCD within the doubly open flavour hadronic system. Though there exist many theoretical models [8–10] to study the hadron properties based on its quark structure, the predictions for low-lying states are off by 60 – 90 MeV with respect to the respective experimental values. Moreover the issue related to the hyperfine and fine structure splitting of the mesonic states; their intricate dependence with the constituent quark masses and the running strong coupling constant are still unresolved. Though the validity of nonrelativistic models is very well established and significantly successful for the description of heavy quarkonia, disparities exist in the the description of meson containing light flavour quarks or antiquarks.

For any successful attempt to understand these states not only be able to satisfactorily predict the mass spectra but also be able to predict their decay properties. For better predictions of the decay widths, many models have incorporated additional contributions such as radiative and higher order QCD corrections [12, 22–25]. Thus, in this paper we make an attempt to study properties like mass spectrum, decay constants and other decay properties of the D meson based on a relativistic Dirac formalism. We investigate the heavy-light mass spectra of D meson in this framework with Martin like confinement potential as in the case of D_s mesons studied recently [26].

Along with the mass spectra, the pseudoscalar decay constants of the heavy-light mesons have also been estimated in the context of many QCD-motivated approximations. The predictions of such methods spread over a wide range of values [27, 28]. It is important thus to have reliable estimate of the decay constant as it is an important parameter in many weak processes such as quark mixing, CP violation, etc. The leptonic decay of charged meson is another important annihilation channel through the exchange of virtual W boson. Though this annihilation process is rare, but they have clear experimental signatures due to the presence of highly energetic leptons in the final state. The leptonic decays of mesons entails an appropriate representation of the initial state of the decaying vector mesons in

terms of the constituent quark and antiquark with their respective momenta and spin. The bound constituent quark and antiquark inside the meson are in definite energy states having no definite momenta. However one can find the momentum distribution amplitude for the constituent quark and antiquark inside the meson just before their annihilation to a lepton pair. Thus, it is appropriate to compute the leptonic branching ratio and compare our result with the experimental values as well as with the predictions based on other models.

2 Theoretical Framework

The quark confining interaction of meson is considered to be produced by the non-perturbative multigluon mechanism and this mechanism is unfeasible to estimate theoretically from first principles of QCD. On the other hand there exist ample experimental support for the quark structure of hadrons. This is the origin of phenomenological models which are proposed to understand the properties of hadrons and quark dynamics at the hadronic scale. To first approximation, the confining part of the interaction is believed to provide the zeroth-order quark dynamics inside the meson through the quark Lagrangian density

$$\mathcal{L}_q^0(x) = \bar{\psi}_q(x) \left[\frac{i}{2} \gamma^\mu \vec{\partial}_\mu - V(r) - m_q \right] \psi_q(x). \quad (2.1)$$

In this context for the present study, we assume that the constituent quark - antiquark inside a meson is independently confined by an average potential of the form [26, 29]

$$V(r) = \frac{1}{2}(1 + \gamma_0)(\lambda r^{0.1} + V_0) \quad (2.2)$$

In the stationary case, the spatial part of the quark wave functions $\psi(\vec{r})$ satisfies the Dirac equation given by

$$[\gamma^0 E_q - \vec{\gamma} \cdot \vec{P} - m_q - V(r)]\psi_q(\vec{r}) = 0. \quad (2.3)$$

The solution of Dirac equation can be written as two component (positive and negative energies in the zeroth order) form as

$$\psi_{nlj}(r) = \begin{pmatrix} \psi_{nlj}^{(+)} \\ \psi_{nlj}^{(-)} \end{pmatrix} \quad (2.4)$$

where

$$\psi_{nlj}^{(+)}(\vec{r}) = N_{nlj} \begin{pmatrix} ig(r)/r \\ (\sigma \cdot \hat{r})f(r)/r \end{pmatrix} \mathcal{Y}_{ljm}(\hat{r}) \quad (2.5)$$

$$\psi_{nlj}^{(-)}(\vec{r}) = N_{nlj} \begin{pmatrix} i(\sigma \cdot \hat{r})f(r)/r \\ g(r)/r \end{pmatrix} (-1)^{j+m_j-l} \mathcal{Y}_{ljm}(\hat{r}) \quad (2.6)$$

and N_{nlj} is the overall normalization constant. The normalized spin angular part is expressed as

$$\mathcal{Y}_{ljm}(\hat{r}) = \sum_{m_l, m_s} \langle l, m_l, \frac{1}{2}, m_s | j, m_j \rangle Y_l^{m_l} \chi_{\frac{1}{2}}^{m_s} \quad (2.7)$$

Table 1. The fitted model parameters for the D systems

System Parameters	D
Quark mass (in GeV)	$m_{u/d} = 0.003$ and $m_c = 1.27$
Potential strength (λ)	$2.2903 + B \text{ GeV}^{\nu+1}$
V_0	- 2.6711 GeV
Centrifugal parameter (B)	$(n * 0.153) \text{ GeV}^{-1}$ for $l = 0$ $((n + l) * 0.1267) \text{ GeV}^{-1}$ for $l \neq 0$
σ ($j - j$ coupling strength)	0.0055 GeV^3 for $l = 0$ 0.0946 GeV^3 for $l \neq 0$

Here the spinor $\chi_{\frac{1}{2}m_s}$ are eigenfunctions of the spin operators,

$$\chi_{\frac{1}{2}\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_{\frac{1}{2}-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.8)$$

The reduced radial part $g(r)$ of the upper component and $f(r)$ of the lower component of Dirac spinor $\psi_{nlj}(r)$ are the solutions of the equations given by

$$\frac{d^2 g(r)}{dr^2} + \left[(E_D + m_q)[E_D - m_q - V(r)] - \frac{\kappa(\kappa + 1)}{r^2} \right] g(r) = 0 \quad (2.9)$$

and

$$\frac{d^2 f(r)}{dr^2} + \left[(E_D + m_q)[E_D - m_q - V(r)] - \frac{\kappa(\kappa - 1)}{r^2} \right] f(r) = 0 \quad (2.10)$$

It can be transformed into a convenient dimensionless form given as [30]

$$\frac{d^2 g(\rho)}{d\rho^2} + \left[\epsilon - \rho^{0.1} - \frac{\kappa(\kappa + 1)}{\rho^2} \right] g(\rho) = 0 \quad (2.11)$$

and

$$\frac{d^2 f(\rho)}{d\rho^2} + \left[\epsilon - \rho^{0.1} - \frac{\kappa(\kappa - 1)}{\rho^2} \right] f(\rho) = 0 \quad (2.12)$$

where $\rho = (r/r_0)$ is a dimensionless variable with the arbitrary scale factor chosen conveniently as

$$r_0 = \left[(m_q + E_D) \frac{\lambda}{2} \right]^{-\frac{10}{21}}, \quad (2.13)$$

and ϵ is a corresponding dimensionless energy eigenvalue defined as

$$\epsilon = (E_D - m_q - V_0)(m_q + E_D)^{\frac{1}{21}} \left(\frac{2}{\lambda} \right)^{\frac{20}{21}} \quad (2.14)$$

Here, it is suitable to define a quantum number κ by

$$\kappa = \begin{cases} -(\ell + 1) = -(j + \frac{1}{2}) & \text{for } j = \ell + \frac{1}{2} \\ \ell = +(j + \frac{1}{2}) & \text{for } j = \ell - \frac{1}{2} \end{cases} \quad (2.15)$$

Equations (2.11) and (2.12) now can be solved numerically [31] for each choice of κ .

The solutions $g(\rho)$ and $f(\rho)$ are normalized to get

$$\int_0^\infty (f_q^2(\rho) + g_q^2(\rho)) d\rho = 1. \quad (2.16)$$

The wavefunction for a $D(c\bar{q})$ meson now can be constructed using Eqn (2.5) and (2.6) and the corresponding mass of the quark-antiquark system can be written as

$$M_{Q\bar{q}}(n_1 l_1 j_1, n_2 l_2 j_2) = E_D^Q + E_D^{\bar{q}} \quad (2.17)$$

where $E_D^{Q/\bar{q}}$ are obtained using Eqn. (2.14) and (2.15) which include the centrifugal repulsion of the centre of mass. For the spin triplet (vector) and spin singlet (pseudoscalar) state, the choices of (j_1, j_2) are $((l_1 + \frac{1}{2}), (l_2 + \frac{1}{2}))$ and $((l_{1,2} + \frac{1}{2}), (l_{2,1} - \frac{1}{2}))$ respectively. The previous work of independent quark model within the Dirac formalism by [26, 29] has been extended here by incorporating the spin-orbit and tensor interactions of the confined one gluon exchange potential (COGEP) [32, 33], in addition to the j-j coupling of the quark-antiquark. Finally, the mass of the specific $^{2S+1}L_J$ states of $Q\bar{q}$ system is expressed as

$$M_{2S+1L_J} = M_{Q\bar{q}}(n_1 l_1 j_1, n_2 l_2 j_2) + \langle V_{Q\bar{q}}^{j_1 j_2} \rangle + \langle V_{Q\bar{q}}^{LS} \rangle + \langle V_{Q\bar{q}}^T \rangle \quad (2.18)$$

The spin-spin part is defined here as

$$\langle V_{Q\bar{q}}^{j_1 j_2}(r) \rangle = \frac{\sigma \langle j_1 j_2 JM | \hat{j}_1 \cdot \hat{j}_2 | j_1 j_2 JM \rangle}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})} \quad (2.19)$$

where σ is the $j-j$ coupling constant. The expectation value of $\langle j_1 j_2 JM | \hat{j}_1 \cdot \hat{j}_2 | j_1 j_2 JM \rangle$ contains the (j_1, j_2) coupling and the square of Clebsch-Gordan coefficients. The tensor and spin-orbit parts of confined one-gluon exchange potential (COGEP) [32, 33] are given as

$$V_{Q\bar{q}}^T(r) = -\frac{\alpha_s}{4} \frac{N_Q^2 N_{\bar{q}}^2}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})} \otimes \lambda_Q \cdot \lambda_{\bar{q}} \left(\left(\frac{D_1''(r)}{3} - \frac{D_1'(r)}{3r} \right) S_{Q\bar{q}} \right) \quad (2.20)$$

where $S_{Q\bar{q}} = [3(\sigma_Q \cdot \hat{r})(\sigma_{\bar{q}} \cdot \hat{r}) - \sigma_Q \cdot \sigma_{\bar{q}}]$ and $\hat{r} = \hat{r}_Q - \hat{r}_{\bar{q}}$ is the unit vector in the direction of \vec{r} and

$$\begin{aligned} V_{Q\bar{q}}^{LS}(r) = & \frac{\alpha_s}{4} \frac{N_Q^2 N_{\bar{q}}^2}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})} \frac{\lambda_Q \cdot \lambda_{\bar{q}}}{2r} \\ & \otimes [\vec{r} \times (\hat{p}_Q - \hat{p}_{\bar{q}}) \cdot (\sigma_Q + \sigma_{\bar{q}})] (D_0'(r) + 2D_1'(r)) \\ & + [\vec{r} \times (\hat{p}_Q + \hat{p}_{\bar{q}}) \cdot (\sigma_i - \sigma_j)] (D_0'(r) - D_1'(r)) \end{aligned} \quad (2.21)$$

where α_s is the strong coupling constant and it is computed as

$$\alpha_s = \frac{4\pi}{(11 - \frac{2}{3} n_f) \log \left(\frac{E_Q^2}{\Lambda_{QCD}^2} \right)} \quad (2.22)$$

with $n_f = 3$ and $\Lambda_{QCD} = 0.150$ GeV. In Eqs. (2.21) the spin-orbit term has been split into symmetric $(\sigma_Q + \sigma_q)$ and anti-symmetric $(\sigma_Q - \sigma_q)$ terms.

Table 2. S-wave D ($c\bar{s}$) spectrum (in MeV).

nL	J^P	State	$M_{Q\bar{q}}$	$\langle V_{Q\bar{q}}^{j_1 j_2} \rangle$	Present	Experiment		[34] ^a	[35] ^b	[13] ^c	[36] ^d	[1] ^e	[19] ^f	QSR ^g
						Meson	Mass[2]							
1S	1 ⁻	1^3S_1	2009.54	0.99	2010.53	D^*	2010.28 ± 0.13		2010	2018	2010	2038	2013	2000 ± 20 [17]
	0 ⁻	1^1S_0	1869.57	-2.58	1867.00	D	1864.86 ± 0.13		1871	1865	1867	1874	1890	1900 ± 30 [17]
2S	1 ⁻	2^3S_1	2605.29	0.57	2605.86	$D^*(2600)$	$2608.7 \pm 2.4 \pm 2.5$ [37]	2639	2632	2639	2636	2645	2708	2612 ± 6 [14]
	0 ⁻	2^1S_0	2523.05	-1.33	2521.72	$D(2550)$	$2539.4 \pm 4.5 \pm 6.8$ [37]	2567	2581	2598	2555	2583	2642	2539 ± 8 [14]
3S	1 ⁻	3^3S_1	3147.50	0.39	3147.89			3125	3096	3110		3111	3103	
	0 ⁻	3^1S_0	3087.21	-0.90	3086.31			3065	3062	3087		3068	3064	
4S	1 ⁻	4^3S_1	3662.99	0.29	3663.28				3482	3514			3395	
	0 ⁻	4^1S_0	3614.22	-0.66	3613.56				3452	3498			3299	

^a Semi-relativistic model

^b Quasi potential Approach

^c Relativistic quark-antiquark potential (Coulomb plus power) model

^d Non-relativistic constituent quark model

^e Relativistic quark model

^f Lattice QCD [LQCD]

^g QCD Sum Rule [QSR]

We have adopted the same parametric form of the confined gluon propagators which are given by [32, 33]

$$D_0(r) = \left(\frac{\alpha_1}{r} + \alpha_2 \right) \exp(-r^2 c_0^2/2) \quad (2.23)$$

and

$$D_1(r) = \frac{\gamma}{r} \exp(-r^2 c_1^2/2) \quad (2.24)$$

with $\alpha_1 = 0.036$, $\alpha_2 = 0.056$, $c_0 = 0.1017$ GeV, $c_1 = 0.1522$ GeV, $\gamma = 0.0139$ as in our earlier study [26]. Other optimized model parameters employed in the present study are listed in Table 1. The current charm quark mass of 1.27 GeV is taken from the PDG (Particle data group)[2]. In the case of $l \neq 0$ orbitally excited states, we find a small variations in the choice of V_0 for the $l = 0$ states due to the centrifugal repulsion from the center of mass of the bound system which is proportional to $(n + l)$. This centrifugal repulsion thus incorporates the centre-of-mass correction.

The computed S-wave masses and other P-wave and D-wave masses of D meson states are listed in Table 2 and Table 3 respectively. A statistical analysis of the sensitivity of the model parameters (i.e. potential strength (λ) and $j - j$ coupling strength σ in the present case) shows about 0.76% variations in the binding energy with 5% changes in the parameters λ and σ . Fig.(1) shows the energy level diagram of D meson spectra along with available experimental results.

Table 3. P-wave and D-wave D ($c\bar{u}$ or $c\bar{d}$) spectrum (in MeV).

nL	J^P	State	$M_{Q\bar{q}}$	$\langle V_{Q\bar{q}}^{j_1 j_2} \rangle$	$\langle V^T \rangle$	$\langle V^{LS} \rangle$	Present	Experiment		[34]	[35]	[13]	[36]	[1]	[19]	[17]
								Meson	Mass [2]							
1P	2^+	1^3P_2	2411.01	8.60	-3.46	52.07	2468.22	$D_2(2460)$	2462.6 ± 0.7		2460	2473	2466	2501	2510	
	1^+	1^3P_1	2411.01	28.68	17.32	-52.07	2404.94	$D_1(2430)$	$2427 \pm 26 \pm 25$		2469	2454	2417	2465	2478	2380 ± 50
	0^+	1^3P_0	2411.01	43.02	-34.65	-104.14	2315.24	$D_0(2400)$	2318 ± 29		2406	2352	2252	2398	2342	2450 ± 30
	1^+	1^1P_1	2312.60	55.34	0	0	2367.94	$D_1(2420)$	2421.3 ± 0.6		2426	2434	2402	2457	2446	
2P	2^+	2^3P_2	2903.96	5.89	-5.57	83.73	2988.02			2965	3012	2971	2971	2957	3084	
	1^+	2^3P_1	2903.96	19.65	27.83	-83.73	2867.72			2960	3021	2951	2926	2952	3055	
	0^+	2^3P_0	2903.96	29.47	-55.67	-167.46	2710.31			2880	2919	2868	2752	2932	2996	
	1^+	2^1P_1	2835.21	36.31	0	0	2871.51			2940	2932	2940	2886	2933	3051	
3P	2^+	3^3P_2	3362.89	4.43	-7.37	110.91	3470.86				3407				3417	
	1^+	3^3P_1	3362.89	14.76	36.85	-110.91	3303.59				3461				3408	
	0^+	3^3P_0	3362.89	22.14	-73.71	-221.83	3089.49				3346				3351	
	1^+	3^1P_1	3309.13	26.81	0	0	3335.94				3365				3338	
1D	3^-	1^3D_3	2839.42	-8.51	-0.02	0.46	2831.34			2840	2971	2834	2811	2833	2870	
	2^-	1^3D_2	2839.42	-25.30	0.08	-0.23	2813.97			2885	2961	2816	2788	2834	2868	
	1^-	1^3D_1	2839.42	-42.91	-0.08	-0.69	2795.74			2870	2913	2873	2804	2816	2850	
	2^-	1^1D_2	2761.19	-1.04	0	0	2760.15	$D(2750)$	$2752.4 \pm 1.7 \pm 2.7$ [37]	2828	2931	2896	2849	2827	2866	
2D	3^-	2^3D_3	3307.69	-6.04	-0.02	0.45	3302.08			3285	3469	3263	3240	3226	3479	
	2^-	2^3D_2	3307.69	-17.94	0.09	-0.22	3289.61				3456	3248	3217	3235	3426	
	1^-	2^3D_1	3307.69	-30.42	-0.09	-0.68	3276.51			3290	3383	3292	3217	3231	3194	
	2^-	2^1D_2	3247.65	-0.72	0	0	3246.93				3403	3312	3260	3225	3401	
3D	3^-	3^3D_3	3753.22	-4.58	-0.03	0.52	3749.14									
	2^-	3^3D_2	3753.22	-13.60	0.10	-0.26	3739.46									
	1^-	3^3D_1	3753.22	-23.06	-0.10	-0.79	3729.27									
	2^-	3^1D_2	3753.22	-0.54	0	0	3703.34									

3 Magnetic (M1) Transitions of Open Charm Meson

Spectroscopic studies led us to compute the decay widths of energetically allowed radiative transitions of the type, $A \rightarrow B + \gamma$ among several vector and pseudoscalar states of D meson. The magnetic transition correspond to spin flip and hence the vector meson decay to pseudoscalar $V \rightarrow P\gamma$ represents a typical M1 transition. Such transitions are experimentally important to the identification of newly observed states. Assuming that such transitions are single vertex processes governed mainly by photon emission from independently confined quark and antiquark inside the meson, the S-matrix elements in the rest frame of the initial meson is written in the form

$$S_{BA} = \left\langle B\gamma \left| -ie \int d^4x T \left[\sum_q e_q \bar{\psi}_q(x) \gamma^\mu \psi_q(x) A_\mu(x) \right] \right| A \right\rangle. \quad (3.1)$$

The common choice of the photon field $A_\mu(x)$ is made here in Coulomb-gauge with $\epsilon(k, \lambda)$ as the polarization vector of the emitted photon having energy momentum ($k_0 = |\mathbf{k}|$, \mathbf{k}) in

the rest frame of A. The quark field operators find a possible expansions in terms of the complete set of positive and negative energy solutions given by Eqs. (2.5) and (2.6) in the form

$$\Psi_q(x) = \sum_{\zeta} \left[b_{q\zeta} \psi_{q\zeta}^{(+)}(r) \exp(-iE_{q\zeta}t) + b_{q\zeta}^{\dagger} \psi_{q\zeta}^{(-)}(r) \exp(iE_{q\zeta}t) \right] \quad (3.2)$$

where the subscript q stands for the quark flavor and ζ represents the set of Dirac quantum numbers. Here $b_{q\zeta}$ and $b_{q\zeta}^{\dagger}$ are the quark annihilation and the antiquark creation operators corresponding to the eigenmodes ζ . After some standard calculations (the details of calculations can be found in Refs. [38, 39] and [40]), the S-matrix elements can be expressed as

$$S_{BA} = i\sqrt{\left(\frac{\alpha}{k}\right)} \delta(E_B + k - E_A) \sum_{q,m,m'} \langle B | \left[J_{m'm}^q(k, \lambda) b_{qm'}^{\dagger} b_{qm} - \tilde{J}_{mm'}^{\tilde{q}}(k, \lambda) \tilde{b}_{qm'}^{\dagger} \tilde{b}_{qm} \right] | A \rangle \quad (3.3)$$

Here $E_A = M_A$, $E_B = \sqrt{k^2 + M_B^2}$ and (m, m') are the possible spin quantum numbers of the confined quarks corresponding to the ground state of the mesons. We have

$$J_{m'm}^q(k, \lambda) = e_q \int d^3r \exp(-i\vec{k} \cdot \vec{r}) [\bar{\psi}_{qm'}^{(+)}(r) \vec{\gamma} \cdot \vec{\epsilon}(k, \lambda) \psi_{qm}^{(+)}(r)] \quad (3.4)$$

$$\tilde{J}_{mm'}^{\tilde{q}}(k, \lambda) = e_q \int d^3r \exp(-i\vec{k} \cdot \vec{r}) [\bar{\psi}_{qm}^{(-)}(r) \vec{\gamma} \cdot \vec{\epsilon}(k, \lambda) \psi_{qm'}^{(-)}(r)]. \quad (3.5)$$

One can reduce the above equations to simple forms as

$$J_{m'm}^q(k, \lambda) = -i \mu_q(k) [\chi_m^{\dagger}(\vec{\sigma} \cdot \vec{K}) \chi_m], \quad (3.6)$$

and

$$\tilde{J}_{mm'}^{\tilde{q}}(k, \lambda) = i \mu_q(k) [\tilde{\chi}_m^{\dagger}(\vec{\sigma} \cdot \vec{K}) \tilde{\chi}_m] \quad (3.7)$$

where $\vec{K} = \vec{k} \times \vec{\epsilon}(k, \lambda)$. Eqs. (3.3) further simplified to get

$$S_{BA} = i\sqrt{\left(\frac{\alpha}{k}\right)} \delta(E_B + k - E_A) \sum_{q,m,m'} \langle B | \mu_q(k) \left[\chi_m^{\dagger} \vec{\sigma} \cdot \vec{K} \chi_m b_{qm'}^{\dagger} b_{qm} + \tilde{\chi}_m^{\dagger} \vec{\sigma} \cdot \vec{K} \tilde{\chi}_m \tilde{b}_{qm'}^{\dagger} \tilde{b}_{qm} \right] | A \rangle \quad (3.8)$$

where $\mu_q(k)$ is expressed as

$$\mu_q(k) = \frac{2e_q}{k} \int_0^{\infty} j_1(kr) f_q(r) g_q(r) dr \quad (3.9)$$

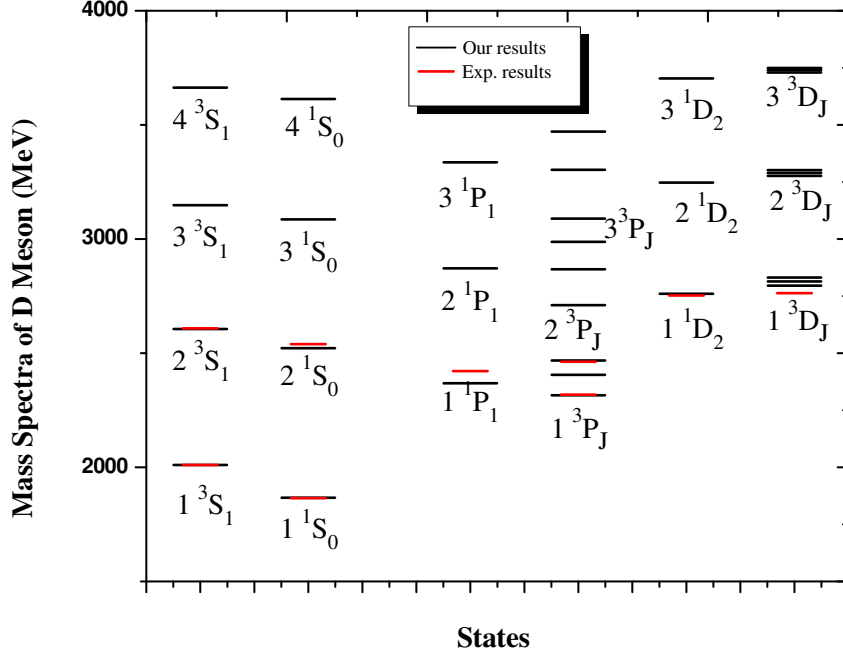


Figure 1. D meson spectra.

where $j_1(kr)$ is the spherical Bessel function and the energy of the outgoing photon in the case of a vector meson undergoing a radiative transition to its pseudoscalar state, for instance, $D^* \rightarrow D\gamma$ is given by

$$k = \frac{M_{D^*}^2 - M_D^2}{2M_{D^*}} \quad (3.10)$$

The relevant transition magnetic moment is expressed as

$$\mu_{D^+D^{*+}}(k) = \frac{1}{3}[2\mu_c(k) - \mu_d(k)], \quad (3.11)$$

$$\mu_{D^0D^{*0}}(k) = \frac{2}{3}[2\mu_c(k) + \mu_u(k)], \quad (3.12)$$

Now, the Magnetic (M1) transition width of $D^* \rightarrow D\gamma$ can be obtained as

$$\Gamma_{D^{*+} \rightarrow D^+\gamma} = \frac{4\alpha}{3}k^3 |\mu_{D^+D^{*+}}(k)|^2 \quad (3.13)$$

$$\Gamma_{D^{*0} \rightarrow D^0\gamma} = \frac{4\alpha}{3}k^3 |\mu_{D^0D^{*0}}(k)|^2 \quad (3.14)$$

The computed transition widths of low lying S-wave states are tabulated in Table 6 and are compared with other model predictions.

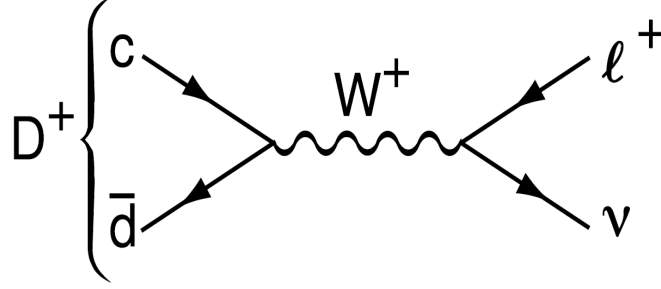


Figure 2. Feynman diagram for leptonic decay ($M \rightarrow l\bar{\nu}_l$)

Table 4. Comparison of Center of Mass in D meson in MeV.

M_{CW}	Present	[13]	[35]	Exp.
$\overline{1S}$	1974.64	1979.75	1975.25	1973.92
$\overline{2S}$	2584.82	2628.75	2619.25	2591.37
$\overline{3S}$	3132.49	3104.25	3087.50	
$\overline{4S}$	3650.85	3510.00	3474.50	
$\overline{1^3P_J}$	2430.13	2453.22	2457.00	2434.66
$\overline{1P}$	2414.58	2448.42	2449.25	2431.22
$\overline{2^3P_J}$	2917.06	2952.88	3004.66	
$\overline{2P}$	2905.67	2949.66	2986.50	

4 Decay constant of D meson

The decay constant of a meson is an important parameter in the study of leptonic or non-leptonic weak decay processes. The decay constant (f_p) of pseudoscalar state is obtained by parameterizing the matrix elements of weak current between the corresponding meson and the vacuum as [41]

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 c | P_\mu \rangle = i f_p P^\mu \quad (4.1)$$

It is possible to express the quark-antiquark eigenmodes in the ground state of the meson in terms of the corresponding momentum distribution amplitudes. Accordingly, eigenmodes, $\psi_A^{(+)}$ in the state of definite momentum p and spin projection s'_p can be expressed as

$$\psi_A^{(+)} = \sum_{s'_p} \int d^3p G_q(p, s'_p) \sqrt{\frac{m}{E_p}} U_q(p, s'_p) \exp(i\vec{p} \cdot \vec{r}) \quad (4.2)$$

where $U_q(p, s'_p)$ is the usual free Dirac spinors.

Table 5. Mass splitting in D meson in MeV.

Splitting	Present	[43]	[13]	[35]	Exp.
$1^3S_1 - 1^1S_0$	143.53	$130.8 \pm 3.2 \pm 1.8$	153	139	140.65 ± 0.1
$2^3S_1 - 2^1S_0$	84.14		41	51	
$3^3S_1 - 3^1S_0$	61.58		23	34	
$4^3S_1 - 4^1S_0$	49.72		16	30	
$D_0(2400)-\overline{1S}$	340.60	$266.9 \pm 17.3 \pm 3.7$	372.25	430.75	347.0 ± 29
$D_1(2420)-\overline{1S}$	393.30	$399.1 \pm 13.5 \pm 5.6$	454.25	450.75	451.6 ± 0.6
$D_1(2430)-\overline{1S}$	430.30	$525.2 \pm 19.4 \pm 7.4$	474.25	493.75	456.0 ± 40
$D_2(2460)-\overline{1S}$	493.58	$577.1 \pm 20.3 \pm 8.1$	493.25	484.75	491.4 ± 1.0

In the relativistic quark model, the decay constant can be expressed through the meson wave function $G_q(p)$ in the momentum space [39, 42]

$$f_P = \left(\frac{3|I_p|^2}{2\pi^2 M_p J_p} \right)^{\frac{1}{2}} \quad (4.3)$$

Here M_p is mass of the pseudoscalar meson and I_p and J_p are defined as

$$I_p = \int_0^\infty dp p^2 A(p) [G_{q1}(p) G_{q2}^*(-p)]^{\frac{1}{2}} \quad (4.4)$$

$$J_p = \int_0^\infty dp p^2 [G_{q1}(p) G_{q2}^*(-p)] \quad (4.5)$$

respectively. Where,

$$A(p) = \frac{(E_{p1} + m_{q1})(E_{p2} + m_{q2}) - p^2}{[E_{p1} E_{p2} (E_{p1} + m_{q1})(E_{p2} + m_{q2})]^{\frac{1}{2}}} \quad (4.6)$$

and $E_{p_i} = \sqrt{k_i^2 + m_{q_i}^2}$.

The computed decay constants of D meson from $1S$ to $4S$ states are tabulated in Table 7. Present result for $1S$ state is compared with experimental as well as other model predictions. There are no model predictions available for comparison of the decay constants of the $2S$ to $4S$ states.

5 Leptonic Decay of D Meson

Charged mesons produced from a quark and anti-quark can decay to a charged lepton pair when these objects annihilate via a virtual W^\pm boson as given in Fig.(2). Though the leptonic decays of open flavour mesons belong to rare decay [47, 48], they have clear experimental signatures due to the presence of highly energetic lepton in the final state. And such decays are very clean due to the absence of hadrons in the final state [49]. The leptonic width of D meson is computed using the relation given by [2]

$$\Gamma(D \rightarrow l^+ \nu_l) = \frac{G_F^2}{8\pi} f_D^2 |V_{cd}|^2 m_l^2 \left(1 - \frac{m_l^2}{M_D^2} \right)^2 M_D \quad (5.1)$$

Table 6. Magnetic (M1) transition of Open Charm Meson

Process	k (MeV)		Γ (keV)					
	Present	[13]	<i>Present</i>	PDG [2]	[13]	[44]	[45]	[46]
$(1S)D^{*0} \rightarrow D^0\gamma$	138.38	147.00	1.2614	< 945	0.339	23.94	10.25	11.5
$(2S)D^{*0} \rightarrow D^0\gamma$	82.84	41.00	0.0289		0.007			
$(3S)D^{*0} \rightarrow D^0\gamma$	60.99	23.00	0.0026		0.001			
$(3S)D^{*0} \rightarrow D^0\gamma$	49.46	16.00	0.0004		0.000			
$(1S)D^{*+} \rightarrow D^+\gamma$	138.38	147.00	0.0837	< 198	0.339	0.94	1.36	1.04
$(2S)D^{*+} \rightarrow D^+\gamma$	82.84	41.00	0.0020		0.007			
$(3S)D^{*+} \rightarrow D^+\gamma$	60.99	23.00	0.0002		0.001			
$(3S)D^{*+} \rightarrow D^+\gamma$	49.46	16.00	0.0000		0.000			

Table 7. Pseudoscalar decay constant (f_P) of D systems (in MeV).

	f_P			
	1S	2S	3S	4S
Present	202.57	292.14	351.066	392.49
PDG [2]	206.7 ± 8.9			
[CPP_ν] [12]	154			
[$QCDSR$] [50]	204 ± 6			
[RPM] [51]	208 ± 21			
[$QCDSR$] [16]	206.2 ± 7.3			
[$LQCD$] [52]	197 ± 9			
[$LQCD$] [53]	218.9 ± 11.3			
[$LFQM$] [54]	206.0 ± 8.9			
[$QCDSR$] [55]	208 ± 11			
[$RBSM$] [27]	229 ± 43			
[$LQCD$] [56]	207 ± 11			
[$LQCD$] [57]	208 ± 3			

[CPP_ν]- Coloumb plus power potential Model

[$QCDSR$]- QCD sum rule.

[RPM]- Relativistic potential Model.

[$LQCD$]- Lattice QCD.

[$LFQM$]- Light front quark model.

[$RBSM$]- Relativistic Bethe-Salpeter Method.

in complete analogy to $\pi^+ \rightarrow l^+\nu$. These transitions are helicity suppressed; i.e., the amplitude is proportional to m_l , the mass of the lepton l . The leptonic widths of D (1^1S_0) meson are obtained from Eqs.(5.1) where the predicted values of the pseudoscalar decay constant f_D along with the masses of M_D and the PDG value for $V_{cd} = 0.230$ are used.

The leptonic widths for separate lepton channel are computed for the choices of $m_{l=\tau,\mu,e}$. The branching ratio of these leptonic widths are then obtained as

$$BR = \Gamma(D \rightarrow l^+ \nu_l) \times \tau \quad (5.2)$$

where τ is the experimental lifetime of the respective D meson state. The computed leptonic widths are tabulated in Table 8 along with other model predictions as well as with the available experimental values. Our results are found to be in accordance with the reported experimental values.

6 Hadronic Decays of D Meson

Study of flavour changing decays of heavy flavour quarks are useful for determining the parameters of the Standard Model and for testing phenomenological models which include strong effects. The interpretation of the hadronic decays of c -meson within a hadronic state is complicated by the effects of strong interaction and by its interplay with the weak interaction. The hadronic decays of heavy mesons can be understood in this model and we assume that Cabibbo favored hadronic decays proceed via the basic process, ($c \rightarrow q + u + \bar{d}; q \in s, d$) and the decay widths are given by [41]

$$\Gamma(D^0 \rightarrow K^- \pi^+) = C_f \frac{G_F^2 |V_{cs}|^2 |V_{ud}|^2 f_\pi^2}{32 \pi M_{D_s}^3} \times [\lambda(M_D^2, M_{K^-}^2, M_\pi^2)]^{\frac{3}{2}} |f_+(q^2)| \quad (6.1)$$

for $q = s$ and

$$\Gamma(D^0 \rightarrow K^+ \pi^-) = C_f \frac{G_F^2 |V_{cd}|^2 |V_{us}|^2 f_\pi^2}{32 \pi M_{D_s}^3} \times [\lambda(M_D^2, M_{K^+}^2, M_\pi^2)]^{\frac{3}{2}} |f_+(q^2)| \quad (6.2)$$

for $q = d$. Here, C_f is the color factor and ($|V_{cs}|, |V_{cd}|, |V_{us}|$) are the CKM matrices. f_π is the decay constant of π meson and its value is taken as 0.130 GeV. Here, $f_+(q^2)$ is the form factor and the factor $\lambda(M_D^2, M_{K^+}^2, M_\pi^2)$ can be computed as

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - xy - yz - zx \quad (6.3)$$

The renormalized color factor without the interference effect due to QCD is given by $(C_A^2 + C_B^2)$. The coefficient C_A and C_B are further expressed as [41]

$$C_A = \frac{1}{2}(C_+ + C_-) \quad (6.4)$$

$$C_B = \frac{1}{2}(C_+ - C_-) \quad (6.5)$$

where

$$C_+ = 1 - \frac{\alpha_s}{\pi} \log \left(\frac{M_W}{m_c} \right) \quad (6.6)$$

Table 8. The leptonic decay width and leptonic Branching Ratio (BR) of D meson.

Process	$\Gamma(D^+ \rightarrow l\bar{\nu}_l)$ (keV)		BR				
	Present	[42]	Present	[13]	[42]	[12]	Experiment [2]
$D^+ \rightarrow \tau^+\nu_\tau$	6.157×10^{-10}	4.72×10^{-13}	9.73×10^{-4}	1.05×10^{-3}	7.54×10^{-4}	1.5×10^{-3}	$< 1.2 \times 10^{-3}$
$D^+ \rightarrow \mu^+\nu_\mu$	2.433×10^{-10}	1.79×10^{-13}	3.84×10^{-4}	4.3×10^{-3}	2.87×10^{-4}	2.2×10^{-4}	3.82×10^{-4}
$D^+ \rightarrow e^+\nu_e$	5.706×10^{-15}		9.02×10^{-9}	1.00×10^{-8}		0.5×10^{-8}	$< 8.8 \times 10^{-6}$

and

$$C_- = 1 + 2 \frac{\alpha_s}{\pi} \log \left(\frac{M_W}{m_c} \right) \quad (6.7)$$

where M_W is the mass of W meson.

Consequently, the form factors $f_\pm(q^2)$ correspond to the D final state are related to the Isgur Wise function as [41]

$$f_\pm(q^2) = \xi(\omega) \frac{M_D \pm M_\phi}{2\sqrt{M_D M_\phi}} \quad (6.8)$$

The Isgur Wise function, $\xi(\omega)$ can be evaluated according to the relation given by [58]

$$\xi(\omega) = \frac{2}{\omega - 1} \left\langle j_0 \left(2 E_q \sqrt{\frac{\omega - 1}{\omega + 1}} r \right) \right\rangle \quad (6.9)$$

where E_q is the binding energy of decaying meson and ω is given by,

$$\omega = \frac{M_D^2 + M_{(K^+, K^-)}^2 - q^2}{2M_D M_{(K^+, K^-)}} \quad (6.10)$$

For a good approximation the form factor $f_-(q^2)$ do not contribute to the decay rate, so we have neglected here. The heavy flavour symmetry provides model-independent normalization of the weak form factors $f_\pm(q^2)$ either at $q = 0$ or $q = q_{max}$ and we have applied $q = q_{max}$ in Eqs. (6.1) and (6.2) for hadronic decay. From the computed exclusive semileptonic and hadronic decay widths, the Branching ratios are obtained as

$$BR = \Gamma \times \tau \quad (6.11)$$

here the lifetime (τ) of D ($\tau_{D^+} = 1.040 \text{ ps}^{-1}$ and $\tau_{D^0} = 0.410 \text{ ps}^{-1}$) is taken as the world average value reported by Particle Data Group (PDG-2012)[2]. The decay widths and their branching ratios are listed in Table 9 along with the known experimental and other theoretical predictions for comparison.

7 Mixing Parameters of $D - \bar{D}$ Oscillation

A different D^0 decay channel [61–65] has been reported by three experimental groups as evidence of $D^0 - \bar{D}^0$ oscillation. We discuss here the mass oscillation of the neutral open

Table 9. The Hadronic decay width and Branching Ratio (BR) of D meson.

Process	$\Gamma(D)$ (keV)	BR		
	Present	Present	[59]	Experiment [2]
$D^0 \rightarrow K^- \pi^+$	6.153×10^{-14}	3.835×10^{-2}	$(3.91 \pm 0.17)\%$	$(3.91 \pm 0.08)\%$ [60]
$D^0 \rightarrow K^+ \pi^-$	1.716×10^{-16}	1.069×10^{-4}	$(1.12 \pm 0.05) \times 10^{-4}$	$(1.48 \pm 0.07) \times 10^{-4}$ [60]

charm meson and the integrated oscillation rate using our spectroscopic parameters deduced from the present study. In the standard model, the transitions $D^0 - \bar{D}^0$ and $\bar{D}^0 - D^0$ occur through the weak interaction. The neutral D meson mix with their antiparticle leading to oscillations between the mass eigenstates [2]. In the following, we adopt the notation introduced in [2], and assume CPT conservation in our calculations. If CP symmetry is violated, the oscillation rates for meson produced as D^0 and \bar{D}^0 can differ, further enriching the phenomenology. The study of CP violation in D^0 oscillation may lead to an improved understanding of possible dynamics beyond the standard model [66–68].

The time evolution of the neutral D -meson doublet is described by a Schrödinger equation with an effective 2×2 Hamiltonian given by [41, 69]

$$i \frac{d}{dt} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \left(M - \frac{i}{2} \Gamma \right) \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} \quad (7.1)$$

where the \mathbf{M} and Γ matrices are Hermitian, and are defined as

$$\left(M - \frac{i}{2} \Gamma \right) = \left[\begin{pmatrix} M_{11}^q & M_{12}^{q*} \\ M_{12}^q & M_{11}^q \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11}^q & \Gamma_{12}^{q*} \\ \Gamma_{12}^q & \Gamma_{11}^q \end{pmatrix} \right]. \quad (7.2)$$

CPT invariance imposes

$$M_{11} = M_{22} \equiv M, \Gamma_{11} = \Gamma_{22} \equiv \Gamma. \quad (7.3)$$

The off-diagonal elements of these matrices describe the dispersive and absorptive parts of $D^0 - \bar{D}^0$ mixing [70]. The two eigenstates D_1 and D_2 of the effective Hamiltonian matrix $(M - \frac{i}{2} \Gamma)$ are given by

$$|D_1\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} (p|D^0\rangle + q|\bar{D}^0\rangle), \quad (7.4)$$

$$|D_2\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} (p|D^0\rangle - q|\bar{D}^0\rangle). \quad (7.5)$$

The corresponding eigenvalues are

$$\lambda_{D_1} \equiv m_1 - \frac{i}{2} \Gamma_1 = \left(M - \frac{i}{2} \Gamma \right) + \frac{q}{p} \left(M_{12} - \frac{i}{2} \Gamma_{12} \right), \quad (7.6)$$

$$\lambda_{D_2} \equiv m_2 - \frac{i}{2} \Gamma_2 = \left(M - \frac{i}{2} \Gamma \right) - \frac{q}{p} \left(M_{12} - \frac{i}{2} \Gamma_{12} \right), \quad (7.7)$$

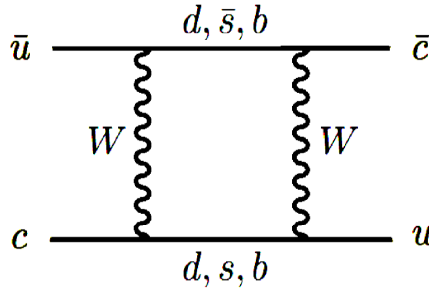


Figure 3. $D^0 - \bar{D}^0$ mixing

where $m_1(m_2)$ and $\Gamma_1(\Gamma_2)$ are the mass and width of $D_1(D_2)$, respectively, and

$$\frac{q}{p} = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \right)^{1/2} \quad (7.8)$$

From Eqs. (7.6) and (7.7), one can get the differences in mass and width which are given as

$$\Delta m \equiv m_2 - m_1 = -2\text{Re} \left[\frac{q}{p} (M_{12} - \frac{i}{2}\Gamma_{12}) \right], \quad (7.9)$$

$$\Delta \Gamma \equiv \Gamma_2 - \Gamma_1 = -2\text{Im} \left[\frac{q}{p} (M_{12} - \frac{i}{2}\Gamma_{12}) \right]. \quad (7.10)$$

The calculation of the dispersive and absorptive parts of the box diagrams yields the following expressions for the off-diagonal element of the mass and decay matrices; for example, if s/\bar{s} as the intermediate quark state then [71],

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_{D^0} m_{D^0} B_{D^0} f_{D^0}^2}{12\pi^2} S_0(m_s^2/m_W^2) (V_{us}^* V_{cs})^2 \quad (7.11)$$

$$\Gamma_{12} = \frac{G_F^2 m_c^2 \eta'_{D^0} m_{D^0} B_{D^0} f_{D^0}^2}{8\pi} [(V_{us}^* V_{cs})^2] \quad (7.12)$$

where G_F is the Fermi constant, m_W is the W boson mass, m_c is the mass of c quark, m_{D^0} , f_{D^0} and B_{D^0} are the D^0 mass, weak decay constant and bag parameter respectively. The known function $S_0(x_q)$ can be approximated very well by $0.784 x_q^{0.76}$ [72] and V_{ij} are the elements of the CKM matrix [73]. The parameter η_D^0 and η'_{D^0} correspond to the gluonic corrections. The only non-negligible contributions to M_{12} are from box diagrams involving $s(\bar{s}), d(\bar{d}), b(\bar{b})$ intermediate quarks in Fig. (3). The phases of M_{12} and Γ_{12} satisfy

$$\phi_M - \phi_\Gamma = \pi + \mathcal{O} \left(\frac{m_c^2}{m_b^2} \right), \quad (7.13)$$

implying that the mass eigenstates have mass and width differences of opposite signs. This means that, like in the $K^0 - \bar{K}^0$ system, the heavy state is expected to have a smaller decay

width than that of the light state: $\Gamma_1 < \Gamma_2$. Hence, $\Delta\Gamma = \Gamma_2 - \Gamma_1$ is expected to be positive in the Standard Model. Further, the quantity

$$\left| \frac{\Gamma_{12}}{M_{12}} \right| \simeq \frac{3\pi}{2} \frac{m_c^2}{m_W^2} \frac{1}{S_0(m_q^2/m_W^2)} \sim \mathcal{O}\left(\frac{m_q^2}{m_t^2}\right) \quad (7.14)$$

is small, and a power expansion of $|q/p|^2$ yields

$$\left| \frac{q}{p} \right|^2 = 1 + \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin(\phi_M - \phi_\Gamma) + \mathcal{O}\left(\left| \frac{\Gamma_{12}}{M_{12}} \right|^2\right). \quad (7.15)$$

Therefore, considering both Eqs.(7.13) and (7.14), the CP -violating parameter given by

$$1 - \left| \frac{q}{p} \right|^2 \simeq \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \quad (7.16)$$

is expected to be very small: $\sim \mathcal{O}(10^{-3})$ for the $D^0 - \bar{D}^0$ system. In the approximation of negligible CP violation in mixing, the ratio $\Delta\Gamma/\Delta m$ is equal to the small quantity $|\Gamma_{12}/M_{12}|$ of Eqs. (7.14); it is hence independent of CKM matrix elements, *i.e.*, the same for the $D^0 - \bar{D}^0$ system.

Theoretically, the hadron lifetime (τ_{D^0}) is related to Γ_{11} ($\tau_{D^0} = 1/\Gamma_{11}$), while the observable Δm and $\Delta\Gamma$ are related to M_{12} and Γ_{12} as [2]

$$\Delta m = 2|M_{12}| \quad (7.17)$$

and

$$\Delta\Gamma = 2|\Gamma_{12}| \quad (7.18)$$

The gluonic correction can find from by different model like Wilson coefficient and evolution of Wilson coefficient from the new physics scale [68]. We have used values of gluonic correction ($\eta_{D^0} = 0.86$; $\eta'_{D^0} = 0.21$) from [74, 75]. The bag parameter $B_{D^0} = 1.34$ is taken from the lattice result of [76], while the pseudoscalar mass (M_{D^0}) and the pseudoscalar decay constant (f_D) of D mesons are the values obtained from our present study using relativistic independent quark model using Martin like potential. The values of m_s (0.1 GeV), M_W (80.403 GeV) and the CKM matrix elements $V_{cs}(1.006)$ and $V_{us}(0.2252)$ are taken from the Particle Data Group [2]. The resulting mass oscillation parameter Δm are tabulated in Table-10 with latest experimental results.

The integrated oscillation rate (χ_q) is the probability to observe a \bar{D} meson in a jet initiated by a \bar{c} quark. As the mass difference Δm_D is a measure of the frequency of the change from a D^0 into a \bar{D}^0 or vice versa. This change is reflected in either the time-dependent oscillations or in the time-integrated rates corresponding to the di-lepton events having the same sign. The time evolution of the neutral states from the pure $|D^0_{phys}\rangle$ or $|\bar{D}^0_{phys}\rangle$ state at $t = 0$ is given by

$$|D^0_{phys}(t)\rangle = g_+(t)|D^0\rangle + \frac{q}{p}g_-(t)|\bar{D}^0\rangle, \quad (7.19)$$

$$|\bar{D}^0_{phys}(t)\rangle = g_+(t)|\bar{D}^0\rangle + \frac{p}{q}g_-(t)|D^0\rangle, \quad (7.20)$$

which means that the flavor states remain unchanged (g_+) or oscillate into each other (g_-) with time-dependent probabilities proportional to

$$g_+(t) = e^{\frac{-\Gamma t}{2}} e^{-i t m_{D^0}} \cos(t\Delta m/2), \quad (7.21)$$

$$g_-(t) = e^{\frac{-\Gamma t}{2}} e^{-i t m_{D^0}} \sin(t\Delta m/2). \quad (7.22)$$

Starting at $t = 0$ with initially pure D^0 , the probability for finding a $D^0(\bar{D}^0)$ at time $t \neq 0$ is given by $|g_+(t)|^2 (|g_-(t)|^2)$. Taking $|q/p| = 1$, one gets

$$|g_{\pm}(t)|^2 = \frac{1}{2} e^{\frac{-\Gamma_D t}{2}} [1 \pm \cos(t\Delta m)]. \quad (7.23)$$

Conversely, from an initially pure \bar{D}^0 at $t = 0$, the probability for finding a $\bar{D}^0(D^0)$ at time $t \neq 0$ is also given by $|g_+(t)|^2 (|g_-(t)|^2)$. The oscillation of D^0 or \bar{D}^0 as shown by Eqs. (7.23) give Δm directly. Integrating $|g_{\pm}(t)|^2$ from $t = 0$ to $t = \infty$, we get

$$\int_0^\infty |g_{\pm}(t)|^2 dt = \frac{1}{2} \left[\frac{1}{\Gamma} \pm \frac{\Gamma}{\Gamma^2 + (\Delta m)^2} \right] \quad (7.24)$$

where $\Gamma = \Gamma_D = (\Gamma_1 + \Gamma_2)/2$. The ratio

$$r_o = \frac{D^0 \leftrightarrow \bar{D}^0}{D^0 \leftrightarrow D^0} = \frac{\int_0^\infty |g_-(t)|^2 dt}{\int_0^\infty |g_+(t)|^2 dt} = \frac{x^2}{2 + x^2}, \quad (7.25)$$

$$\text{where } x_q = x = \frac{\Delta m}{\Gamma} = \Delta m \tau_D, \quad y_q = \frac{\Delta \Gamma}{2\Gamma} = \frac{\Delta \Gamma \tau_D}{2},$$

$$\chi_q = \frac{x_q^2 + y_q^2}{2(x_q^2 + 1)}, \quad (7.26)$$

reflects the change of pure D^0 into a \bar{D}^0 , or vice versa.

The time-integrated mixing rate relative to the time-integrated right-sign decay rate for semileptonic decays [2] is

$$R_M = \int_0^\infty r(t) dt = \int_0^\infty |g_-(t)|^2 \left| \frac{q}{p} \right|^2 dt \quad (7.27)$$

$$R_M = \int_0^\infty \frac{e^{-t}}{4} (x^2 + y^2) t^2 \left| \frac{q}{p} \right|^2 dt \simeq \frac{1}{2} (x^2 + y^2) \quad (7.28)$$

In the Standard Model, CP violation in charm mixing is small and $|q/p| \approx 1$.

For the present estimation of these mixing parameters x_q , y_q and χ_q , we employ our predicated Δm values and the experimental average lifetime of PDG [2] of the D -meson.

Table 10. Mixing Parameters x_q , y_q , χ_q and R_M of D mesons

	$\Delta M(GeV)$	x_q	y_q	χ_q	R_M
Present	8.255×10^{-15}	5.14×10^{-3}	6.02×10^{-3}	3.13×10^{-5}	3.13×10^{-5}
[77]		$(0.80 \pm 0.29)\%$	$(0.33 \pm 0.24)\%$		$0.864 \pm 0.311 \times 10^{-4}$
[78]					$0.13 \pm 0.22 \pm 0.20 \times 10^{-3}$
[79]					$0.04^{+0.7}_{-0.6} \times 10^{-3}$
[80]					$0.02 \pm 0.47 \pm 0.14 \times 10^{-3}$

8 Results and Discussion

We have studied here the mass spectra and decay properties of the D meson in the framework of relativistic independent quark model. Our computed D meson spectral states are in good agreement with the reported PDG values of known states. The predicted masses of S-wave D meson state 2^3S_1 (2605.86 MeV) and 2^1S_0 (2521.72 MeV) are in very good agreement with the respective experimental results of $2608 \pm 2.4 \pm 2.5$ MeV [37] and $2539.4 \pm 4.5 \pm 6.8$ MeV [37] by BABAR Collaboration. The expected results of other S-wave excited states of D meson are also in good agreement with other reported values [13, 34–36]. The predicted P-wave D meson states, 1^3P_2 (2468.22 MeV), 1^3P_1 (2404.94 MeV), 1^3P_0 (2315.24 MeV) and 1^1P_1 (2367.94 MeV) are in good agreement with experimental [2] results of 2462.6 ± 0.7 MeV, $2427 \pm 26 \pm 25$ MeV, 2318 ± 29 MeV and 2421.3 ± 0.6 MeV respectively. The 1^1D_2 (2760.15) are very close with the experimental result of $2752.4 \pm 1.7 \pm 2.7$ MeV [37] and we predict its J^P value to be 2^- . We have also compared Lattice QCD and QCD sum rule results with our predicted results where the numerical values in Table 3 for Lattice QCD results are extracted from the energy level diagram available in [19]. With reference to the available experimental masses of D -mesonic states, we observe that the LQCD predictions [19] are off by standard deviation of ± 58.52 and that by QCD sum rule [14, 17] predictions are off by ± 59.22 , while predicted calculations show the standard deviation of ± 21.88 .

In the relativistic Dirac formalism, the spin degeneracy is primarily broken therefore to compare the spin average mass, we employ the relation as

$$M_{CW} = \frac{\sum_J (2J+1) M_J}{\sum_J (2J+1)} \quad (8.1)$$

The spin average or the center of weight masses M_{CW} are calculated from the known values of the different meson states and are compared with other model predictions [35] and [13] in Table 4. The table also contains the different spin dependent contributions for the observed state.

The precise experimental measurements of the masses of D meson states provided a real test for the choice of the hyperfine and the fine structure interactions adopted in the

study of D meson spectroscopy. Recent study of D meson mass splitting in lattice QCD [LQCD] [43] using 2 ± 1 flavor configurations generated with the Clover-Wilson fermion action by the PACS-CS collaboration [43] has been listed for comparison. Present results as seen in Table 9 are in very good agreement with the respective experimental values over the lattice results [43]. In this Table, the present results on an average, are in agreement with the available experimental value within 12% variations, while the lattice QCD predictions [43] show 30% variations.

The magnetic transitions (M1) can probe the internal charge structure of hadrons, and therefore they will likely play an important role in determining the hadronic structures of D meson. The present M1 transitions widths of D meson states as listed in Table 6 are in accordance with the model prediction of [45] while the upper bound provided by PDG [2] is very wide. We do not find any theoretical predictions for M1 transition width of excited states for comparison. Thus we only look forward to see future experimental support to our predictions.

The calculated pseudoscalar decay constant (f_P) of D meson is listed in Table (7) along with other model predictions as well as experimental results. The value of $f_D(1S) = 202.57$ MeV obtained in our present study is in very good agreement with other theoretical predictions for $1S$ state. The predicted f_D for higher S-wave states are found to increase with energy. However, there are no experimental or theoretical values available for comparison. Another important property of D meson studied in the present case is the leptonic decay widths. The present branching ratios for $D \rightarrow \tau \bar{\nu}_\tau$ (9.73×10^{-4}) and $D \rightarrow \mu \bar{\nu}_\mu$ (3.846×10^{-4}) are in accordance with the experimental results ($< 1.2 \times 10^{-2}$) and (3.82×10^{-4}) respectively over other theoretical predictions vide Table 8. Large experimental uncertainty in the electron channel make it difficult for any reasonable conclusion.

The Cabibbo favoured hadronic branching ratio $\text{BR}(D^0 \rightarrow K^- \pi^+)$ and $\text{BR}(D^0 \rightarrow K^+ \pi^-)$ obtained respectively as 3.835% and 1.069×10^{-4} are in very good agreement with Experimental values [60] of $3.91 \pm 0.08\%$ and $(1.48 \pm 0.07) \times 10^{-4}$ respectively.

We obtained the CP violation parameter in mixing $|q/p|$ (0.9996) in this case and D^0 and \bar{D}^0 decays shows no evidence for CP violation and provides the most stringent bounds on the mixing parameters. The mixing parameter x_q , y_q , and mixing rate (R_M) are very good agreement with BaBar, Belle and other Collaboration as shown in Table 10. However, due to larger uncertainty in the experimental values make difficult for us to draw a continuous remark on this mixing parameter. Thus, the present study of the mixing parameters of neutral open charm meson is found to be one of the successful attempt to extract the effective quark-antiquark interaction in the case of heavy-light flavour mesons. Thus the present study is an attempt to indicate the importance of spectroscopic (strong interaction) parameters in the weak decay processes.

Finally we look forward to see future experimental support in favour of many of our

predictions on the spectral states and decay properties of the open charm meson.

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References

- [1] Yuan Sun, Xiang Liu, and Takayuki Matsuki, Newly observed $D_J(3000)^{+,0}$ and $D_J^*(3000)^0$ as 2P states in D meson family, Phys. Rev. **D 88** (2013) 094020.
- [2] K. A. Olive et al., Review of Particle Physics (Particle Data Group), Chin. Phys. **C 38**, 090001 (2014), J. Beringer et al., Review of Particle Physics (Particle Data Group), Phys. Rev. **D 86** (2012) 010001 .
- [3] Z.G. Wang, Is $D_s(2700)$ a charmed tetraquark state ?, Chin. Phys. **C 32** (2008) 797.
- [4] J. Vijande, A. Valcarce, F. Fernandez, Multiquark description of the $D_{sJ}(2860)$ and $D_{sJ}(2700)$, Phys. Rev. **D 79** (2009) 037501.
- [5] Qian Wanga, Christoph Hanhart and Qiang Zhao, Meson loop singularity, exotic states' productions and decays near threshold, Proceedings of Charm2013; arXiv: 1311.2401v1[hep-ph].
- [6] Zhi-Feng Sun, Jie-Sheng Yu and Xiang Liu, Newly observed D(2550), D(2610), and D(2760) as 2S and 1D charmed mesons, Phys. Rev. **D 82** (2010) 111501.
- [7] P. Abreu et al., First evidence for a charm radial excitation, $D^{*'}(DELPHI\ Collaboration)$, Phys. Lett. **B 426** (1998) 231 .
- [8] S. Godfrey and N. Isgur, Mesons in a relativized quark model with chromodynamics, Phys. Rev. **D 32** (1985) 189.
- [9] S Godfrey and R Kokoski, Properties of P-wave mesons with one heavy quark, Phys. Rev. **D 43** (1991) 1679.
- [10] M. Di Pierro, E. Eichten, Excited heavy-light systems and hadronic transitions, Phys. Rev. **D 64** (2001) 114004.
- [11] A F Falk and T Mehen, Excited heavy mesons beyond leading order in the heavy quark expansion, Phys. Rev.D **53** (1996) 231.
- [12] Bhavin Patel and P C Vinodkumar, Decay properties of D and D_s mesons in coulomb plus power potential (CPP_ν), Chin. Phys. **C 34** (2010) 1497; arXiv:0908.2212v1 [hep-ph].
- [13] N. Devlani and A K Rai, Mass Spectrum and Decay Properties of D Meson, Int. J. Theor. Phys. **52** (2013) 2196.
- [14] P. Gelhausen, A. Khodjamiriana, A. A. Pivovarov, D. Rosenthal, Radial excitations of heavy-light mesons from QCD sum rules, Eur. Phys. J. **C 74** (2014) 2979.
- [15] P. Gelhausen, A. Khodjamirian, A. A. Pivovarov, and D. Rosenthal, Decay constants of heavy-light vector mesons from QCD sum rules, Phys. Rev. **D 88** (2013) 014015.

- [16] W. Lucha, D. Melikhov and S. Simula, OPE, charm-quark mass, and decay constants of D and D_s mesons from QCD sum rules, Phys. Lett. **B 701** (2011) 82.
- [17] A. Hayashigaki and K. Terasaki, Charmed-meson spectroscopy in QCD sum rule, arXiv:hep-ph/0411285v1.
- [18] A. Loz  a, M. E. Bracco, R. D. Matheus, and M. Nielsen, Charmed Scalar Mesons Masses within the QCD Sum Rules Framework, Brazilian Journal of Physics, **37** (2007) 67.
- [19] Graham Moir et al., Excited spectroscopy of charmed mesons from lattice QCD, JHEP **05** (2013) 021.
- [20] Graham Moir et al., Excited D and D_s meson spectroscopy from lattice QCD, PoS (Confinement X) 139.
- [21] P. Dimopoulos et al., Pseudoscalar decay constants f_K/f_π , f_D and f_{D_s} with $N_f = 2 + 1 + 1$ ETMC configurations, PoS (LATTICE 2013) 31.
- [22] A K Rai, B Patel and P C Vinodkumar, Properties of $Q\bar{Q}$ mesons in nonrelativistic QCD formalism, Phys. Rev. **C 78** (2008) 055202.
- [23] D. Ebert, R. N. Faustov and V. O. Galkin, Two-photon decay rates of heavy quarkonia in the relativistic quark model, Mod. Phys. Lett **A 18** (2003) 601.
- [24] J P Lansberg and T N Pham, Two-photon width of η'_c and η_c from heavy-quark spin symmetry, Phys. Rev. **D 74** (2006) 034001; Two-photon width of η_b , η'_b and η''_b from heavy-quark spin symmetry, Phys. Rev. **D 75** (2007) 017501; [arXiv:hep-ph/0804.2180v1].
- [25] C S Kim, T Lee and G L Wang, Annihilation rate of heavy 0^{-+} quarkonium in relativistic Salpeter method, Phys. Lett. **B 606** (2005) 323; [arXiv:hep-ph/0411075].
- [26] Manan Shah, Bhavin Patel and P C Vinodkumar, Mass spectra and decay properties of D_s meson in a relativistic Dirac formalism, Phys. Rev. **D 90** (2014) 014009.
- [27] G. L. Wang, Decay constants of heavy vector mesons in relativistic Bethe-  Salpeter method, Phys. Lett. **B 633** (2006) 492.
- [28] G. Cvetic, C. Kim, G.-L. Wang, and W. Namgung, Decay constants of heavy meson of image state in relativistic Salpeter method, Phys. Lett. **B 596** (2004) 84.
- [29] N. Barik, B. K. Dash, and M. Das, Static properties of the nucleon octet in a relativistic potential model with center-of-mass correction, Phys. Rev. **D 31** (1985) 1652.
- [30] N. Barik, S. N. Jena, Lorentz structure vs relativistic consistency of an effective power-law potential model for quark-antiquark systems, Phys. Rev. **D 26** (1982) 2420.
- [31] Bhavin Patel and P C Vinodkumar, Properties of $Q\bar{Q}$ ($Q \in b, c$) mesons in Coulomb plus power potential (CPP_ν), J. Phys. **G 36** (2009) 035003.
- [32] P C Vinodkumar, K B Vijaya Kumar and S B Khadkikar, Effect of the confined gluons in quark-quark interaction, Pramana J. Phys. **39** (1992) 47.
- [33] S B Khadkikar and K B Vijaya Kumar, NN scattering with exchange of confined gluons, Phys. Lett. **B 254** (1991) 320.
- [34] A. M. Badalian, B. L. G. Bakker, Higher excitations of the D and D_s mesons, Phys. Rev. **D 84** (2011) 034006.
- [35] D. Ebert, R. N. Faustov and V. O. Galkin, Heavy-light meson spectroscopy and Regge trajectories in the relativistic quark model, Eur. Phys. J. **C 66** (2010) 197.

- [36] De-Min Li, Peng-Fei Ji, Bing Ma, The newly observed open-charm states in quark model, Eur. Phys. J. **C 71** (2011) 1582.
- [37] P. del Amo Sanchez et al., Observation of new resonances decaying to $D\pi$ and $D^*\pi$ in inclusive e^+e^- collisions near $\sqrt{s} = 10.58$ GeV (BABAR Collaboration), Phys. Rev. **D 82** (2010) 111101.
- [38] N. Barik, P. C. Dash and A. R. Panda, Radiative decay of mesons in an independent-quark potential model, Phys. Rev. **D 46** (1992) 3856.
- [39] N. Barik, P. C. Dash and A. R. Panda, Leptonic decay of light vector mesons in an independent quark model, Phys. Rev. **D 47** (1993) 1001.
- [40] S. N. Jena, S. Panda and T. C. Tripathy, A static calculation of radiative decay widths of mesons in a potential model of independent quarks, Nucl. Phys. **A 658** (1999) 249.
- [41] Quang Ho-Kim and Pham Xuan-Yem, “The particles and their interactions: Concept and Phenomena” Springer-Verlag (1998).
- [42] Hakan Ciftci and Hüseyin Koru, Meson decay in an independent quark model, Int. J. Mod. Phys. **E 9** (2000) 407.
- [43] Daniel Mohler and R. M. Woloshyn, D and D_s meson spectroscopy, Phys. Rev. **D 84** (2011) 054505.
- [44] S N Jena, S Panda and J N Mohanty, Mesonic M1 transitions in a relativistic potential model of independent quarks, J. Phys. G: Nucl Part. Phys. **24** (1998) 1869.
- [45] Hakan Ciftci and Hüseyin Koru, Radiative decay of light and heavy mesons in an independent quark model, Modern Phys. Lett. **A 16** (2001) 1785.
- [46] D. Ebert, R.N. Faustov, V.O. Galkin, Radiative M1-decays of heavy-light mesons in the relativistic quark model, Phys. Lett. **B 537** (2002) 241.
- [47] Hikasa K et al., Review of Particle Properties, (Particle Data Group) Phys. Rev. **D 45** (1992) S1.
- [48] Rosner J L and Stone S, Decay Constants of Charged Pseudoscalar Mesons, arXiv:hep-ex/0802.1043v1.
- [49] Villa S, Review of Bu leptonic decays, arXiv:hep-ex/0707.0263v1.
- [50] S. Narison, A fresh look into $\bar{m}_{c,b}(\bar{m}_{c,b})$ and precise $f_{D(s)}, B(s)$ from heavy-light QCD spectral sum rules, Phys. Lett. **B 718** (2013) 1321 .
- [51] Mao-Zhi Yang, Wave functions and decay constants of B and D mesons in the relativistic potential model, Eur. Phys. J. **C 72** (2012) 1880.
- [52] B. Blossier et al., OPE, charm-quark mass, and decay constants of D and D_s mesons from QCD sum rules, JHEP **0907** (2009) 043.
- [53] A. Bazavov et al., B- and D-meson decay constants from three-flavor lattice QCD (Fermilab Lattice and MILC Collaborations), Phys. Rev. **D 85** (2012) 114506.
- [54] Chien-Wen Hwang, Analyses of decay constants and light-cone distribution amplitudes for s-wave heavy meson, Phys. Rev. **D 81** (2010) 114024.
- [55] Zhi-Gang Wang, Analysis of the decay constants of the heavy pseudoscalar mesons with QCD sum rules, JHEP **10**(2013) 208; arXiv:1301.1399v3 [hep-ph].

- [56] Eduardo Follana, Precision Lattice Calculation of D and Ds decay constants, Proceedings of the CHARM 2007 Workshop, Ithaca, NY, August 5-8 (2007), arXiv:0709.4628v1 [hep-lat] .
- [57] Heechang Na, Precise Determinations of the Decay Constants of B and D mesons, PoS (Lattice 2012) 102, arXiv:1212.0586v1 [hep-lat].
- [58] M. G. Olsson and Sinia Veseli , Relativistic flux tube model calculation of the Isgur-Wise function, Phys. Rev. **D 51** (1995) 2224.
- [59] Hai-Yang Cheng and Cheng-Wei Chiang, Two-body hadronic charmed meson decays, Phys. Rev. **D 81** (2010) 074021.
- [60] H. Mendez et al., Measurements of D meson decays to two pseudoscalar mesons (CLEO Collaboration), Phys. Rev. **D 81** (2010) 052013.
- [61] B. Aubert et al., Evidence for $D^0-\bar{D}^0$ Mixing (BABAR collaboration), Phys. Rev. Lett. **98** (2007) 211802.
- [62] M. Staric et al., Evidence for $D^0-\bar{D}^0$ Mixing (Belle collaboration), Phys. Rev. Lett. **98** (2007) 211803.
- [63] T. Aaltonen et al., Evidence for $D^0-\bar{D}^0$ Mixing Using the CDF II Detector (CDF collaboration), Phys. Rev. Lett. **100** (2008) 121802.
- [64] B. Aubert et al., Measurement of $D^0-\bar{D}^0$ Mixing from a Time-Dependent Amplitude Analysis of $D^0 \rightarrow K^+\pi^-\pi^0$ Decays (BABAR collaboration), Phys. Rev. Lett. **103** (2009) 211801.
- [65] B. Aubert et al., Measurement of $D^0-\bar{D}^0$ mixing using the ratio of lifetimes for the decays $D^0 \rightarrow K^-\pi^+$ and K^+K^- (BABAR collaboration), Phys. Rev. **D 80** (2009) 071103.
- [66] G. Blaylock, A. Seiden, and Y. Nir, The role of CP violation in $D^0-\bar{D}^0$ mixing, Phys. Lett. B **355** (1995) 555.
- [67] A. A. Petrov, Charm mixing in the standard model and beyond, Int. J. Mod. Phys. **A 21** (2006) 5686.
- [68] E. Golowich, J. A. Hewett, S. Pakvasa, and A. A. Petrov, Implications of $D^0-\bar{D}^0$ mixing for new physics, Phys. Rev. **D 76** (2007) 095009.
- [69] Buchalla G et al., B, D and K decays, Eur. Phys. J. **C 57** (2008) 309.
- [70] Ikaros Bigi, and A. I. Sanda, On $D^0-\bar{D}^0$ mixing and CP violation Phys. Lett. B **171** (1986) 320.
- [71] Buras A J, Slominski W and Steger H, $B^0-\bar{B}^0$ mixing, CP violation and the B-meson decay, Nucl. Phys. **B 245** (1984) 369.
- [72] Takeo Inami and C S Lim , Effects of Superheavy Quarks and Leptons in Low-Energy Weak Processes $K_L \rightarrow \mu\bar{\mu}$, $K^+ \rightarrow p^+\nu\bar{\nu}$ and $K^0 \leftrightarrow \bar{K}^0$, Prog. Theor. Phys **65** (1981) 297 [Erratum ibid. 65 (1981) 1772].
- [73] Kobayashi M and Maskawa K, Prog. Theor. Phys. **49** (1973) 652.
- [74] Monika Blanka et al., Littlest Higgs model with T-parity confronting the new data on $D^0-\bar{D}^0$ mixing, Phys. Lett. **B 657** (2007) 8.
- [75] Stephen Herrlich, Ulrich Nierste, The complete $|\Delta S|=2$ Hamiltonian in the next-to-leading order, Nucl. Phys. **B 419** (1994) 292.
- [76] Buras A J, Phys. Lett **B 566** (2003) 115.

- [77] L.M. Zhang et al., Measurement of $D^0 - \bar{D}^0$ Mixing Parameters in $D^0 \rightarrow K_s \pi^+ \pi^-$ Decays (Belle Collaboration), Phys. Rev. Lett. **99** (2007) 131803.
- [78] U. Bitenc et al., Improved search for $D^0 - \bar{D}^0$ mixing using semileptonic decays at Belle (Belle Collaboration), Phys. Rev. **D 77** (2008) 112003.
- [79] B. Aubert et al., Search for $D^0 - \bar{D}^0$ mixing using doubly flavor tagged semileptonic decay modes, (BABAR Collaboration), Phys. Rev. **D 76** (2007) 014018.
- [80] U. Bitenc et al., Search for $D^0 - \bar{D}^0$ mixing using semileptonic decays at Belle (Belle Collaboration), Phys. Rev. **D 72** (2005) 071101.